for two reasons: (1) It was based on a confusion between the four-color map theorem and a simpler theorem, easily proved, which says that five regions on the plane cannot be mutually contiguous, (2) the true four-color theorem, unproved when I wrote my story, has since been established by computer programs, though not very elegantly. As science fiction, the tale is now as dated as a story about Martians, or about the twilight zone of Mercury. If you are curious, you will find the flawed yarn in Future Tense *(1952), edited by Kendell Crossen and in* Fantasia Mathematica *(1958), edited by Clifton Fadiman.*

DOLORES—a tall, black-haired striptease at Chicago's Purple Hat Club—stood in the center of the dance floor and began the slow gyrations of her Cleopatra number, accompanied by soft Egyptian music from the Purple Hatters. The room was dark except for a shaft of emerald light that played over her filmy costume and smooth, voluptuous limbs.

A veil draped about her head and shoulders was the first to be removed. Dolores was in the act of letting it drift gracefully to the floor when suddenly a sound like the firing of a shotgun came from somewhere above and the nude body of a large man dropped head first from the ceiling. He caught the veil in mid-air with his chin and pinned it to the floor with a dull thump.

Pandemonium reigned.

Jake Bowers, the master of ceremonies, yelled for lights and tried to keep back the crowd. The club's manager, who had been standing by the orchestra watching the floor show, threw a tablecloth over the crumpled figure and rolled it over on its back.

The man was breathing heavily, apparently knocked unconscious by the blow on his chin, but otherwise unharmed. He had a short, neatly trimmed red beard and mustache, and a completely bald head. He was built like a professional wrestler.

With considerable difficulty three waiters succeeded in transporting him to the manager's private office in the back, leaving a roomful of bewildered, near-hysterical men and women gaping at the ceiling and each other, and arguing heatedly about the angle and manner of the man's fall. The only hypothesis with even a slight suggestion of sanity was that he had been tossed high into the air from somewhere on the side of the dance floor. But no one saw the tossing. The police were called.

Meanwhile, in the back office the bearded man recovered consciousness. He insisted that he was Dr. Stanislaw Slapenarski, professor of mathematics at the University of Warsaw, and at present a visiting lecturer at the University of Chicago.

Before continuing this curious narrative, I must pause to confess that I was not an eyewitness of the episode just described, having based my account on interviews with the master of ceremonies and several waiters. However, I did participate in a chain of remarkable events which culminated in the professor's unprecedented appearance.

These events began several hours earlier when members of the Moebius Society gathered for their annual banquet in one of the private dining rooms on the second floor of the Purple Hat Club. The Moebius Society is a small, obscure Chicago organization of mathematicians working in the field of topology, one of the youngest and most mysterious of the newer branches of transformation mathematics. To make clear what happened during the evening, it will be necessary at this point to give a brief description of the subject matter of topology.

Topology is difficult to define in nontechnical terms. An informal way to put it is to say that topology studies the mathematical properties of an object that remain constant regardless of how the object is distorted.

Picture in your mind a doughnut made of soft pliable rubber that can be twisted and stretched as far as you like in any direction. No matter how much this rubber doughnut is distorted, certain properties of the doughnut remain unchanged. For example, it will always retain a hole. In topology the doughnut shape is called a "torus." A soda straw is merely an elongated torus. From a topological point of view a doughnut and a soda straw are identical figures.

Topology is unconcerned with quantitative measurements. It studies only those properties of shape that are unchanged throughout the most radical distortions possible without breaking off pieces of the object and sticking them on again at other spots. If this breaking off were permitted, an object of a given structure could be transformed into an object of any other type of structure, and all original properties would be lost. If the reader will reflect a moment he will soon realize that topology studies the most primitive and fundamental geometrical properties that an object can possess.*

A sample problem in topology may be helpful. Imagine a torus (doughnut) surface made of thin rubber like an inner tube. Now imagine a small hole in the side of this torus. Is it possible to turn the torus inside out through this hole, as you might turn a balloon inside out?

Although many mathematicians of the eighteenth century wrestled with isolated topological problems, one of the first systematic works in the field was done by August Ferdinand Moebius, a Ger-

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* A reader who wants a clearer picture of this new mathematics will find articles on topology in the *Encyclopedia Britannica* (Fourteenth Edition) under *Analysis Situs;* and under *Analysis Situs* in the *Encyclopedia Americana.* There are also readable chapters on elementary topology in two recent books—*Mathematics and the Imagination* by Kasner and Newman, and *What Is Mathematics?* by Courant and Robbins. Slapenarski's published work has not yet been translated from Polish.

man astronomer who taught at the University of Leipzig during the first half of the last century. Until the time of Moebius it was believed that any surface, such as a piece of paper, had two sides. It was the German astronomer who made the disconcerting discovery that if you take a strip of paper, give it a single half-twist, then paste the ends together, the result is a "unilateral" surface—a surface with only *one* side!

If you will trouble to make such a strip (known to topologists as the "Moebius surface") and examine it carefully, you will soon discover that the strip actually does consist of only one continuous side and one continuous edge.

It is hard to believe at first that such a strip can exist, but there it is—a visible, tangible thing that can be constructed in a moment. And it has the indisputable property of one-sidedness, a property it cannot lose no matter how much it is stretched or distorted.*

But back to the story. As an instructor in mathematics at the University of Chicago with a doctor's thesis in topology to my credit, I had little difficulty in securing admittance into the Moebius Society. Our membership was small—only twenty-six men, most of them Chicago topologists but a few from universities in neighboring towns.

We held monthly meetings, rather academic in character, and once a year on November 17 (the anniversary of Moebius' birth) we arranged a banquet at which an outstanding topologist was brought

* The Moebius strip has many terrifying properties. For example, if you cut the strip in half lengthwise, cutting down the center all the way around, the result is not two strips, as might be expected, but one single large strip. But if you begin cutting a third of the way from the side, cutting twice around the strip, the result is one large and one small strip, interlocked. The smaller strip can then be cut in half to yield a single large strip, still interlocked with the other large strip. These weird properties are the basis of an old magic trick with cloth, known to the conjuring profession as the "Afghan bands."

to the city to be our guest speaker.

The banquet always had its less serious aspects, usually in the form of special entertainment. But this year our funds were low and we decided to hold the celebration at the Purple Hat where the cost of the dinner would not be too great and where we could enjoy the floor show after the lecture. We were fortunate in having been able to obtain as our guest the distinguished Professor Slapenarski, universally acknowledged as one of the greatest mathematical minds of the century.

Dr. Slapenarski had been in the city several weeks giving a series of lectures at the University of Chicago on the topological aspects of Einstein's theory of space. As a result of my contacts with him at the university, we had become good friends and I was asked to introduce him at the dinner.

We rode to the Purple Hat together in a taxi, and on the way I begged him to give me some inkling of the content of his address. But he only smiled inscrutably and told me, in his thick Polish accent, to wait and see. He had announced his topic as "The No-Sided Surface"—a topic which aroused such speculation among our members that Dr. Robert Simpson of the University of Wisconsin wrote that he was coming to the dinner, the first meeting he had attended in over a year.*

Simpson is the outstanding authority on topology in the Middle West and the author of several important papers on topology and nuclear physics in which he vigorously attacks several of Slapenarski's major axioms.

The Polish professor and I arrived a little late. After introducing

* Simpson later told me that he had attended the dinner not to hear Slapenarski but to see Dolores.

him to Simpson, then to our other members, we took our seats at the table and I called Slapenarski's attention to our tradition of brightening the banquet with little topological touches. For instance, our napkin rings were silver-plated Moebius strips. Doughnuts were provided with the coffee, and the coffee itself was contained in specially designed cups made in the shape of "Klein bottles."*

After the meal we were served Ballantine's ale, because of the curious trade-mark,** and pretzels in the shapes of the two basic "trefoil" knots.*** Slapenarski was much amused by these details and even made several suggestions for additional topological curiosities, but the suggestions are too complex to explain here.

After my brief introduction, the Polish professor stood up, acknowledged the applause with a smile, and cleared his throat. The room instantly became silent. The reader is already familiar with the professor's appearance—his portly frame, reddish beard, and polished pate. Something in the expression of his face suggested that he had matters of weighty import to disclose.

It would be impossible to give in detail the substance of Slape-

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* Named after Felix Klein, a brilliant German mathematician, Klein's bottle is a completely closed surface, like the surface of a globe, but without inside or outside. It is unilateral like a Moebius strip, but unlike the strip it has no edge. It can be bisected in such a way that each half becomes a Moebius surface. It will hold a liquid. Nothing frightful happens to the liquid.

** This trade-mark is a topological manifold of great interest. Although the three rings are interlocked, no *two* rings are interlocked. In other words, if any one of the rings is removed, the other two rings are completely free of each other. Yet the three together cannot be separated.

*** The trefoil knot is the simplest form of knot that can be tied in a closed curve. It exists in two forms, one a mirror image of the other. Although the two forms are topologically identical, it is impossible to transform one into the other by distortion, an upsetting fact that has caused topologists considerable embarrassment. The study of the properties of knots forms an important branch of topology, though very little is understood as yet about even the simplest knots.

narski's brilliant, highly technical address. But the gist of it was this. Ten years ago, he said, he had been impressed by a statement of Moebius, in one of his lesser known treatises, that there was no theoretical reason why a surface could not lose *both* its sides—to become in other words, a "nonlateral" surface.

Of course, the professor explained, such a surface is impossible to imagine, but so is the square root of minus one or the hypercube of four-dimensional geometry. That a concept is inconceivable has long been recognized as no basis for denying either its validity or usefulness in mathematics and physics.

We must remember, he added, that even the one-sided surface is inconceivable to anyone who has not seen and handled a Moebius strip. Many persons, with well-developed mathematical imaginations, are unable to understand how such a strip can exist even when they have one in hand.

I glanced at Dr. Simpson and thought I detected a skeptical smile curving the corners of his mouth.

Slapenarski continued. For many years, he said, he had been engaged in a tireless quest for a no-sided surface. On the basis of analogy with known types of surfaces he had been able to analyze many of the properties of the no-sided surface. Finally one day—he paused here for dramatic emphasis, sweeping his bright little eyes across the motionless faces of his listeners—he had actually succeeded in constructing a no-sided surface.

His words were like an electric impulse that transmitted itself around the table. Everyone gave a sudden start and shifted his position and looked at his neighbor with raised eyebrows. I noticed that Simpson was shaking his head vigorously. When the speaker walked to the end of the room where a blackboard had been placed, Simpson bent his head and whispered to the man on his left, "It's sheer nonsense. Either Slappy has gone completely slaphappy or he's playing a deliberate prank on all of us."

I think it occurred to the others also that the lecture was a hoax because I noticed several were smiling while the professor chalked some elaborate diagrams on the blackboard.

After a somewhat involved discussion of the diagrams (which I was unable to follow) the professor announced that he would conclude his lecture by constructing one of the simpler forms of the no-sided surface. By now we were all grinning at each other. Dr. Simpson's face had more of a smirk than a grin.

Slapenarski produced from his coat pocket a sheet of pale blue paper, a small pair of scissors, and a tube of paste. He cut the paper into a figure that had a striking resemblance, I thought, to a paper doll. There were five projecting strips or appendages that resembled a head and four limbs. Then he folded and pasted the sheet carefully. It was an intricate procedure. Strips went over and under each other in an odd fashion until finally only two ends projected. Dr. Slapenarski applied a dab of paste to one of these ends.

"Gentlemen," he said, holding up the twisted blue construction and turning it about for all to see, "you are about to witness the first public demonstration of the Slapenarski surface."

He pressed one of the projecting ends against the other.

There was a loud pop, like the bursting of a light bulb, and the paper figure vanished in his hands!

For a moment we were too stunned to move, then with one accord we broke into laughter and applause.

We were convinced, of course, that we were the victims of an elaborate joke. But it had been beautifully executed. I assumed, as did the others, that we had witnessed an ingenious chemical trick with paper—paper treated so it could be ignited by friction or some similar method and caused to explode without leaving an ash.

But I noticed that the professor seemed disconcerted by the laughter, and his face was beginning to turn the color of his beard.

He bowed in an embarrassed way and sat down. The applause subsided slowly.

Falling in with the preposterous mood of the evening we all clustered around to congratulate him warmly on his remarkable discovery. Then the man in charge of arrangements reminded us that a table had been reserved below so those interested in remaining could enjoy some drinks and see the floor show.

The room gradually cleared of everyone except Slapenarski, Simpson, and myself. The two famous topologists were standing in front of the blackboard. Simpson was smiling broadly and gesturing toward one of the diagrams.

"The fallacy in your proof is beautifully concealed, Doctor," he said. "I wonder if any of the others detected it."

The Polish mathematician was not amused.

"There is no fallacy in my proof," he said impatiently.

"Oh, come now, Doctor. Of course there's a fallacy." Simpson touched a corner of the diagram with his thumb. "Those lines can't possibly intersect within the manifold. The intersection is somewhere out here." He waved a hand off to the right.

Slapenarski's face was growing red again.

"I tell you there *is* no fallacy," he repeated, his voice rising. Then slowly, speaking his words carefully and explosively, he went over the proof once more, rapping the blackboard at intervals with his knuckles.

Simpson listened gravely, and at one point interrupted with an objection. The objection was answered. A moment later he raised a second objection. The second objection was answered. I stood aside without saying anything. The discussion was too far above my head.

Then they began to raise their voices. I have already spoken of Simpson's long-standing controversy with Slapenarski over several basic topological axioms. Some of these axioms were now being brought into the argument.

"But I tell you the transformation is *not* bicontinuous and therefore the two sets cannot be homeomorphic," Simpson shouted.

The veins on the Polish mathematician's temples were standing out in sharp relief. "Then suppose you explain to me why my manifold vanished," he yelled.

"It was nothing but a cheap conjuring trick," snorted Simpson. "I don't know how it worked and I don't care. It certainly wasn't because the manifold became nonlateral."

"Oh it wasn't, wasn't it?" Slapenarski said between his teeth. Before I had a chance to intervene he sent his huge fist crashing into the jaw of Dr. Simpson. The Wisconsin professor groaned and dropped to the floor. Slapenarski turned and glared at me wildly.

"Get back, young man," he said. Because he outweighed me by at least one hundred pounds, I got back.

Then I watched in horror what was taking place. With insane fury on his face, Slapenarski knelt beside the limp body and began twisting its arms and legs into fantastic knots. He was, in fact, folding the Wisconsin topologist the way he had folded his piece of paper! Suddenly there was a small explosion, like the backfire of a car, and under the Polish mathematician's hands lay the collapsed clothing of Dr. Simpson.

Simpson had become a nonlateral surface.

Slapenarski stood up, breathing with difficulty and holding in his hands a tweed coat with vest, shirt, and underwear top inside. He opened his hands and let the garments fall on top of the clothing on the floor. Great drops of perspiration rolled down his face. He muttered in Polish, then pounded his fists against his forehead.

I recovered enough presence of mind to move to the entrance of the room, and lock the door. When I spoke my voice sounded weak. "Can he . . . be brought back?"

"I do not know, I do not know," Slapenarski wailed. "I have only begun the study of these surfaces—only just begun. I have no way of knowing where he is. Undoubtedly it is one of the higher dimensions, probably an odd-numbered one. God knows which one."

He grabbed me suddenly by my coat lapels and shook me so violently that a bridge on my upper teeth came loose. "I must go to him," he said. "It is the least I can do—the least."

He sat on the floor and began interweaving his arms and legs.

"Don't stand there like an idiot!" he yelled. "Here—some assistance."

I adjusted my bridge, then helped him twist his right arm under his left leg and back around his head until he was able to grip his right ear. Then his left arm had to be twisted in a similar fashion. "Over, not under," he shouted. It was with difficulty that I was able to force his left hand close enough to his face so he could grasp his nose.

There was another explosive noise, much louder than the sound made by Simpson, and a sudden blast of cold wind across my face. When I opened my eyes I saw the second heap of crumpled clothing.

While I was staring stupidly at the two piles of clothing there was a muffled "pfft" sound behind me. I turned and saw Simpson standing near the wall, naked and shivering. His face was white. Then his knees buckled and he sank to the floor. There were vivid red marks at places where his limbs had been pressed tightly against each other.

I stumbled to the door, unlocked it, and started down the stairway after a strong drink—for myself. I became conscious of a violent hubbub on the dance floor. Slapenarski had, a few moments earlier, completed his sensational dive.

In a back room I found the other members of the Moebius Society and various officials of the Purple Hat Club in noisy, incoherent debate. Slapenarski was sitting in a chair with a tablecloth wrapped around him and holding a handkerchief filled with ice cubes against the side of his jaw.

"Simpson is back," I said. "He fainted but I think he's okay."

"Thank heavens," Slapenarski mumbled.

The officials and patrons of the Purple Hat never understood, of course, what happened that wild night, and our attempts to explain made matters worse. The police arrived, adding to the confusion.

We finally got the two professors dressed and on their feet, and made an escape by promising to return the following day with our lawyers. The manager seemed to think that the club had been the victim of an outlandish plot, and threatened to sue for damages against what he called the club's "refined reputation." As it turned out, the incident proved to be marvelous word-of-mouth advertising and eventually the club dropped the case. The papers heard the story, of course, but promptly dismissed it as an uncouth publicity stunt cooked up by Cody Phanstiehl, the Purple Hat's press agent.

Simpson was unhurt, but Slapenarski's jaw had been broken. I took him to Billings Hospital, near the University of Chicago, and in his room late that night he told me what he thought had happened. Apparently Simpson had entered a higher dimension (very likely the fifth) on level ground.

When he recovered consciousness he unraveled himself and immediately returned to our space as a normal three-dimensional torus with outside and inside surfaces. But Slapenarski had worse luck. He had landed on some sort of slope. There was nothing to see—only a gray, undifferentiated fog—but he had the distinct sensation of rolling down a hill.

He tried to keep a grip on his nose but was unable to maintain it. His right hand slipped free before he reached the bottom of the incline. As a result, he unfolded himself and tumbled back into three-dimensional space and into the middle of Dolores' Egyptian routine.

At any rate that was how Slapenarski had it figured.

He was several weeks in the hospital, refusing to see anyone until the day of his release, when I accompanied him to the Union Station. He caught a train to New York and I never saw him again. He died a few months later of a heart attack in Warsaw. At present Dr. Simpson is in correspondence with Slapenarski's widow in an attempt to obtain his notes on nonlateral surfaces.

Whether these notes will or will not be intelligible to American topologists (assuming we can obtain them) remains to be seen. We have made numerous experiments with folded paper, but so far have produced only commonplace bilateral and unilateral surfaces. Although it was I who helped Slapenarski fold himself, the excitement of the moment erased all details from my mind.

But I shall never forget a remark the great topologist made the night of his accident, just before I left him at the hospital.

"It was fortunate," he said, "that both Simpson and I released our right hand before the left."

"Why?" I asked.

Slapenarski shuddered.

"We would have turned inside out," he said.